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Privacy-Enhancing Technologies

Module 2: Measuring Privacy – Metrics

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Disclaimer: This lecture was prepared in cooperation with Prof. Patricia Arias-Cabarcos and Dr. Javier Parra-Arnau

Competence Center for Applied Security Technology

Outline

- The importance of privacy metrics
- Privacy domains
- Aspects of privacy metrics
- Classification of privacy metrics[1]

[1] Isabel Wagner and David Eckhoff, "Technical Privacy Metrics: A Systematic Survey", ACM Comput. Surv. 51, 3, Article 57, June 2018.

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Privacy and human rights

- Data privacy is the adaptation to the Information Society of the fundamental right to privacy and private life.
- It is included by the United Nations in the ***Universal Declaration of Human Rights*** (1948), in Article 12:

> No one shall be subjected to arbitrary interference with his privacy, family, home or correspondence, nor to attacks upon his honor and reputation. Everyone has the right to the protection of the law against such interference or attacks.

The need for privacy and utility metrics

- Data privacy technologies are about technically enforcing that right in the information society
  - Anonymous-communication networks, anonymous credentials, multiparty computation and oblivious transfer protocols are some examples of general-purpose PETs
- The use of these technologies is not widespread yet
  - are seen as an expensive innovation with unclear benefits
  - frequently come at the expense of system functionality and data distortion (a.k.a. utility)

Privacy: What is promised and really achieved?

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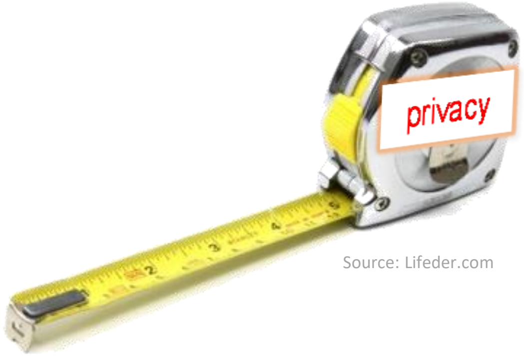
- PETS hide or distort PII
  - (Hopefully) impact on privacy! *(which?)*
  - Impact on utility (in some cases) -> Privacy – utility tradeoff *(s.t.: cost)*

Source: resp. Web pages

ource: census.gov

Privacy and Utility `Metrics‘?

- Quantifiable measures of privacy and utility enable us to
  - assess, compare,
  - improve and optimize privacy-enhancing mechanisms



Source: Lifeder.com

- What is a ‚metric‘?
  - A measure of the extent of inequality
  - Math requires: non-negativity, identity of indiscernibles, symmetry, triangle ineq.
  - Privacy metrics often just measure, and not metrics in the mathematical sense!

- Spectrum of expression
  - Pessimistic / worst-case metrics *(the conventional security view)*
  - Average case
  - Optimistic / best-case metrics

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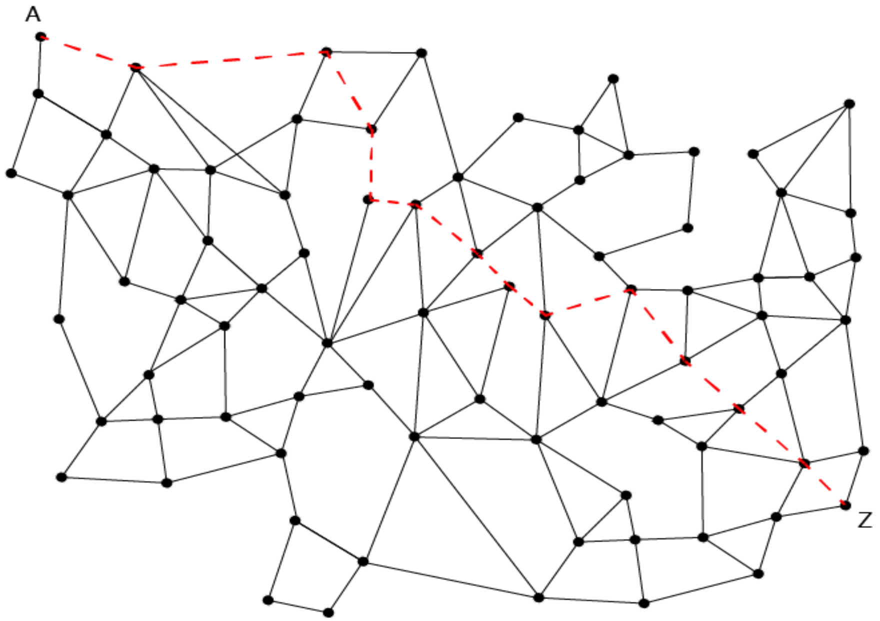
Privacy domains

- Privacy domains are areas where PETs can be applied
- Common privacy domains:
  - Anonymous-communication systems
  - Databases
  - Personalized information systems
  - Location-based services
  - Interaction graph privacy
  - Genome privacy

Privacy domains

- Anonymous-communication systems
  - The goal is to prevent an adversary from linking an outgoing message to its corresponding input message



Privacy domains

- Anonymous-communication systems[2]
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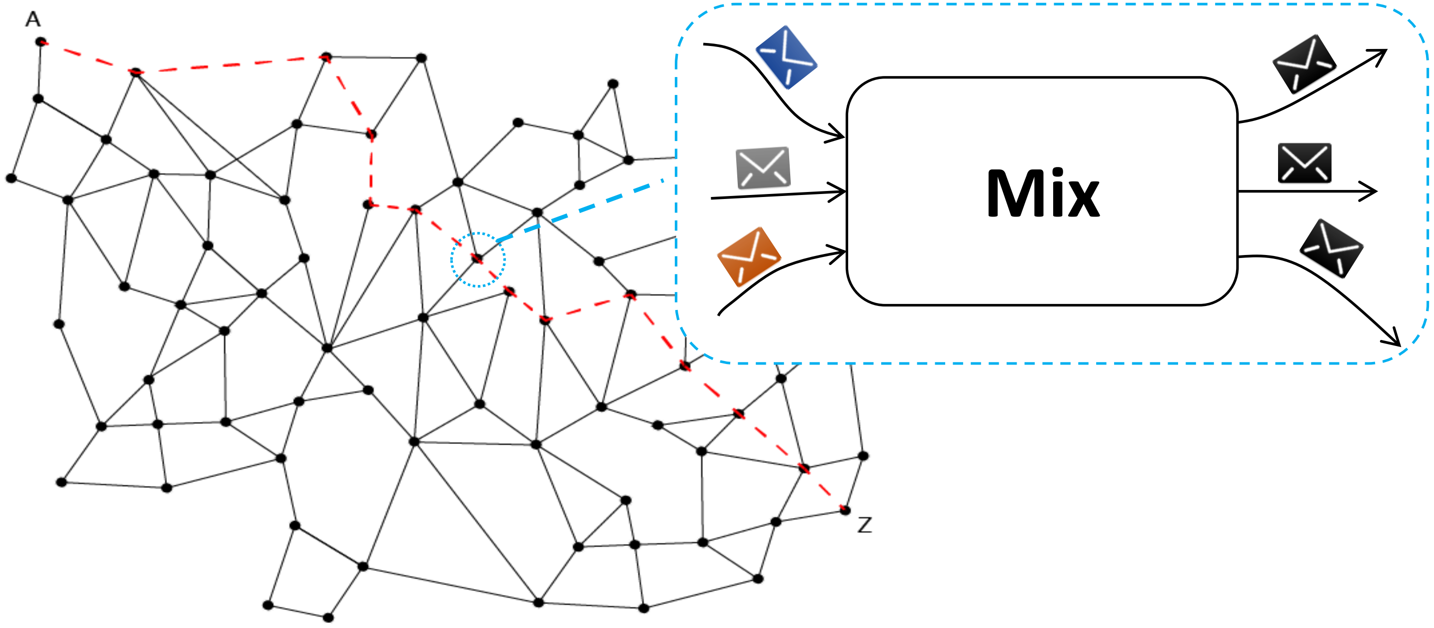


[2] D. Chaum, "Untraceable electronic mail, return addresses, and digital pseudonyms", Commun. ACM, vol. 24, no. 2, pp. 84-88, 1981.

Privacy domains

- Database anonymization
  - E.g., microdata

Identifiers	Key Attributes: Height	Key Attributes: Weight	Confidential Attributes: High Cholesterol
John Smith	5’4”	158	Y
Tang Lee	5’3”	162	Y
Luis Melo	5’6”	161	Y
Anna Frank	5’8”	157	N

Microdata

Privacy domains

- Database anonymization[3]
  - E.g., microdata

Key Attributes		Confidential Attributes
Height	Weight	High Cholesterol
5’4”	158	Y
5’3”	162	Y
5’6”	161	Y
5’8”	157	N

De-identified microdata

[3] L. Sweeney, Uniqueness of Simple Demographics in the U.S. Population, LIDAPWP4. Carnegie Mellon University, Laboratory for International Data Privacy, Pittsburgh, PA, 2000.

Privacy domains

- Database anonymization
  - E.g., microdata

Key Attributes		Confidential Attributes
Height	Weight	High Cholesterol
5’4”	158	Y
5’3”	162	Y
5’6”	161	Y
5’8”	157	N

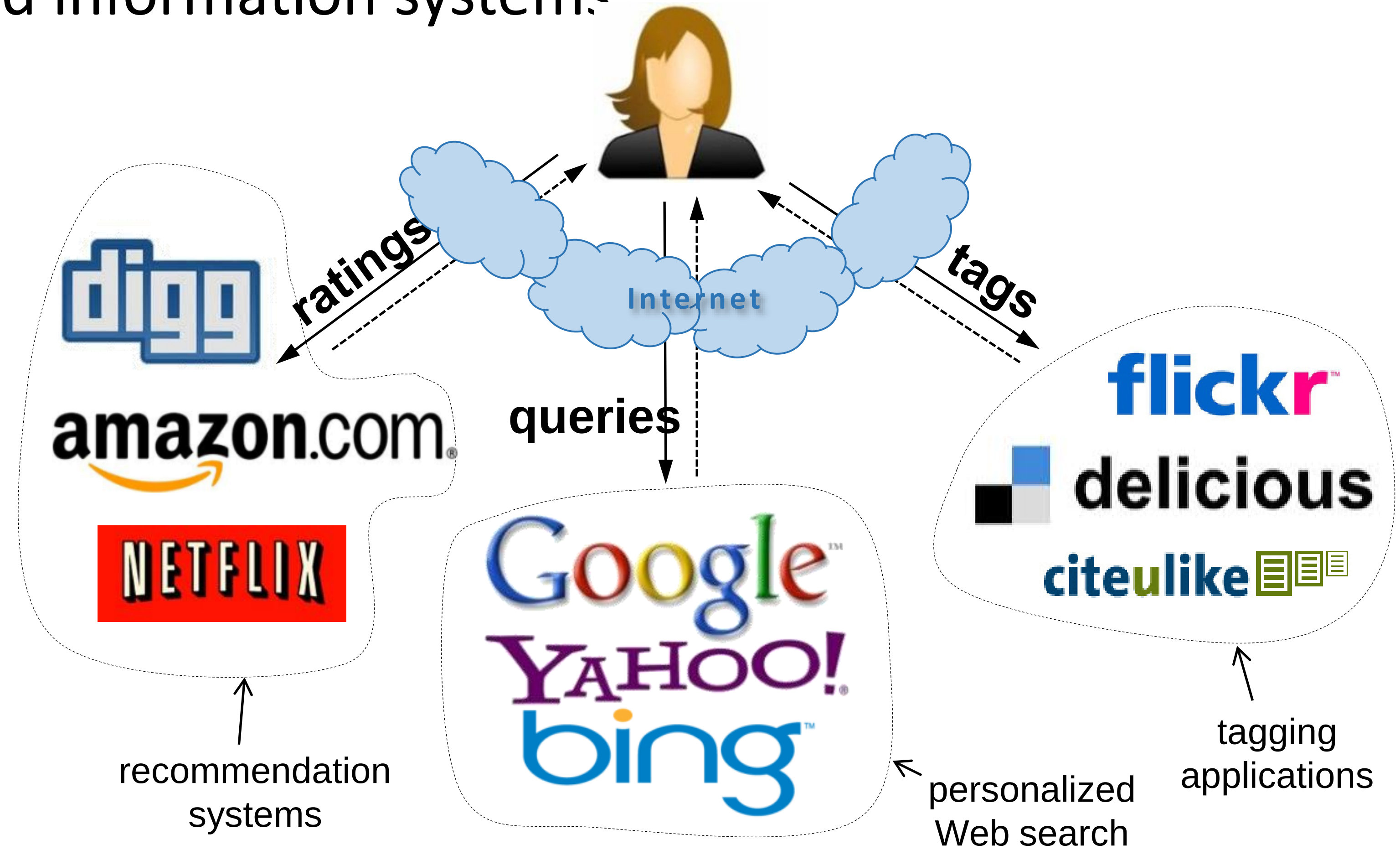
De-identified microdata

	Perturbed Key Attributes		Confidential Attributes
	Height	Weight	High Cholesterol
Aggregated Records	5’5”	160	Y
	5’5”	160	Y
	5’5”	160	Y
	6’0”	155	N

Protected microdata

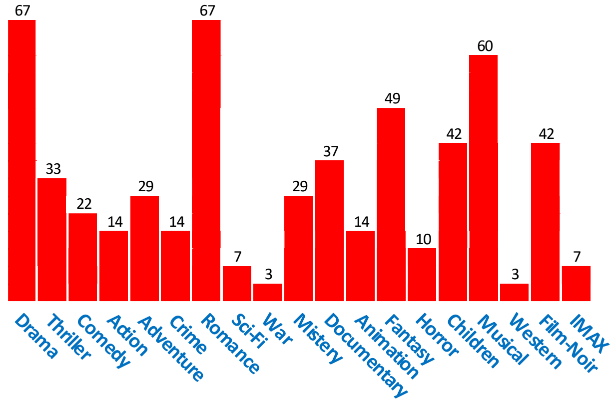
Privacy domains

- Personalized information systems



Privacy domains

- Personalized information systems



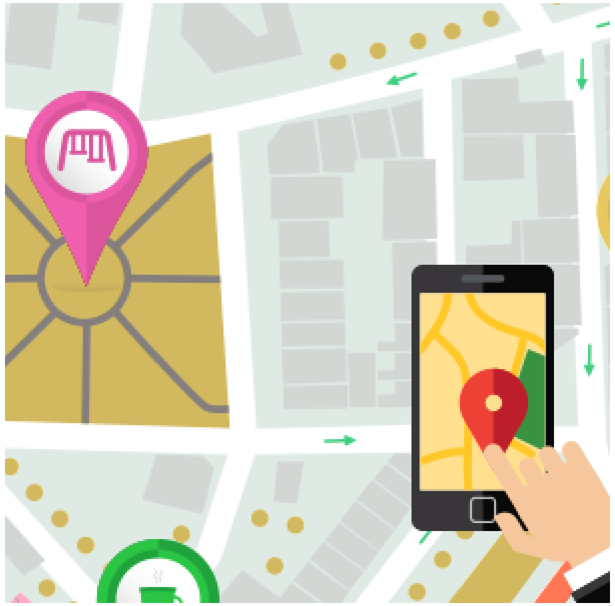
Your categories

Below you can edit the interests and inferred demographics that Google has associated with your cookie:

Category	
Beauty & Fitness – Fitness – Yoga & Pilates	Remove
Hobbies & Leisure – Water Activities – Surf & Swim	Remove
Home & Garden – Home Improvement – House Painting & Finishing	Remove
News – Health News	Remove
People & Society – Family & Relationships – Family – Baby Names	Remove
People & Society – Family & Relationships – Family – Parenting – Baby Care	Remove
Sports – Individual Sports - Cycling	Remove
Sports – Individual Sports – Gymnastics	Remove
Demographics – Age – 25-34	Remove
Demographics – Gender – Female	Remove

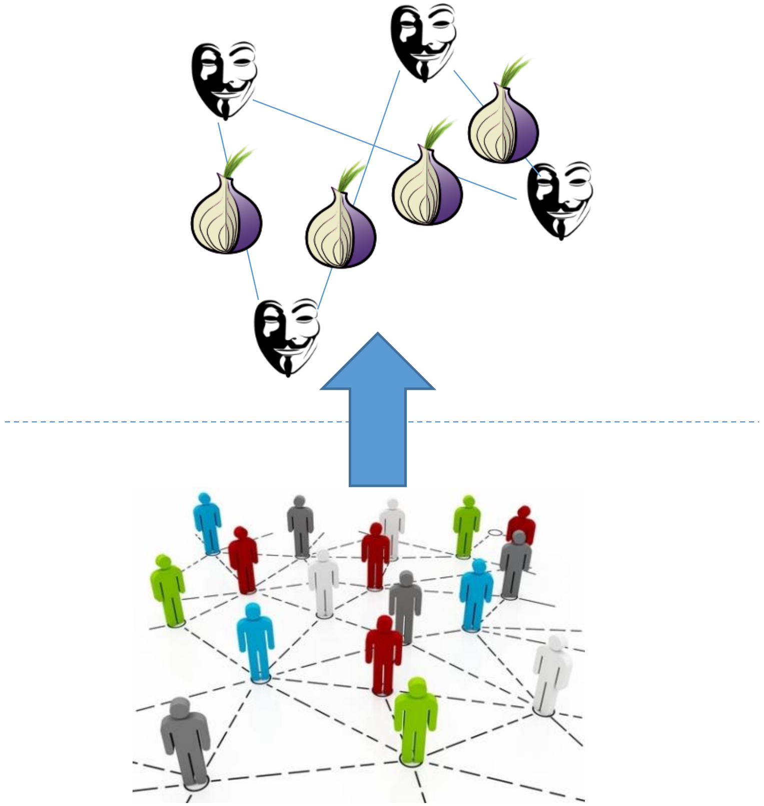
Privacy domains

- Location-based services

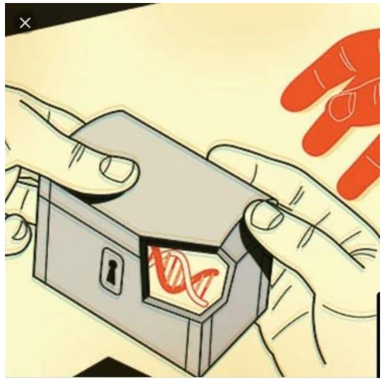


Source: Geospatial World

Interaction Graphs



Genomic Privacy



Source: Scientific American

Outline

- The importance of privacy metrics
- Privacy domains
- Aspects of privacy metrics
- Classification of privacy metrics

Aspects of privacy metrics

- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
  - Output measures

Aspects of privacy metrics

- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
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- Metrics are defined for a specific adversary
- Goals include
  - **identifying** a user
  - user **properties** (interests, preferences, location, etc.)
- Metrics need to be chosen according to that goal

Aspects of privacy metrics

- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
  - Output measures

- Attacker's success depends on its capabilities
- Metrics can only be employed to compare two PETs if they rely on the **same adversary** capabilities
- Taxonomy
  - Local-global
  - Passive-active
  - Internal-External
  - Prior knowledge
  - Resources

Aspects of privacy metrics

- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
  - Output measures

- Which **data** is to be protected? How does the adversary **gain access** to them?
  - Published data
  - Observable data
  - Repurposed data
  - All other data

Aspects of privacy metrics

- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
  - Output measures

- What are **assumptions** about the adversary, protection requirements?
  - Prior knowledge of the adversary
  - Adversary's resources
  - Adversary's estimate
  - Ground truth/true outcome
  - Parameters

Aspects of privacy metrics

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- Although there is a wide variety of privacy metrics, they all share some common features:
  - Adversary goals
  - Adversary capabilities
  - Data sources
  - Input of metric
  - Output measures

- Which **property** is the metric **measuring**?
  - Uncertainty
  - Information gain/loss
  - Data similarity/dissimilarity
  - Indistinguishable
  - Error-based metrics
  - Time-based metrics

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- The importance of privacy metrics
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- Aspects of privacy metrics
- Privacy metrics by class (output)
    - Uncertainty-based
    - Information-gain/loss
    - Estimation error
    - Time-based metrics
    - Data-similarity
    - Indistinguishability-based

1) Uncertainty-based privacy metrics

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- Assume that low uncertainty in the adversary's estimate correlates with low privacy

- The majority of these privacy metrics rely upon information-theoretic quantities (e.g., entropy)
- Origin in anonymous-communication systems

- Examples
  - Anonymity set size[4]
  - Shannon's entropy[5]
  - Normalized Shannon's entropy[5]
  - Inherent privacy[6]
  - Rényi entropy[7]

[4] D. Chaum, "The dining cryptographers problem: unconditional sender and recipient untraceability. J. Cryptol. vol. 1, no. 1, pp. 65–75, March 1988.

[5] C. Diaz, S. Seys, J. Claessens, B. Preneel, "Towards measuring anonymity", Privacy Enhancing Technologies (PET'02). LNCS 2482, pp. 54-68, 2002.

[6] C. Andersson, R. Lundin, "On the fundamentals of anonymity metrics", In Proc. IFIP Int. Summer School on the Future of Identity in the Information Society. Karlstad, Sweden, pp. 325–341, 2008.

[7] S. Clauß, S. Schiffner, "Structuring anonymity metrics", In Proc. ACM Workshop on Digital Identity Management (DIM'06), pp. 55-62, 2006.

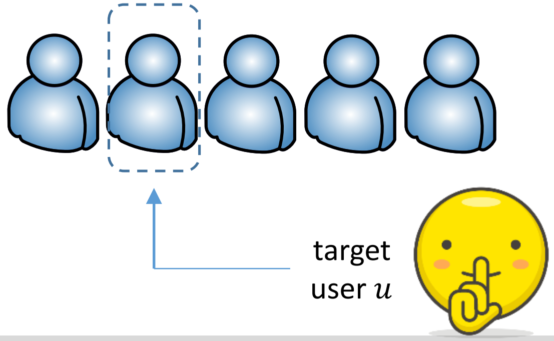
Anonymity set (size)

- Given a target member u, it is defined as the (size of the) set of members the adversary cannot distinguish from u

- The larger the anonymity set, the more anonymity a member is enjoying
- Widely used metric, not only in ACSs

- Simplicity, tractability are positive properties of this metric
- However: it only depends on the number of members in the system

the metric assumes the attacker cannot distinguish any of them



Figures sources: blog.yellowoctopus.com.au, iconscout.com

Shannon's entropy

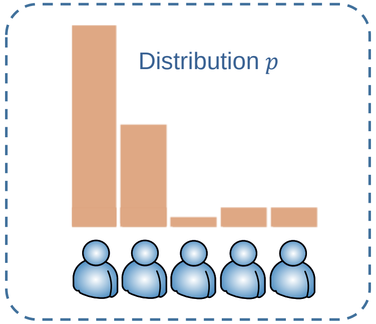
- From information-theory, it measures the uncertainty associated with predicting the outcome of a random variable (r.v.)

- As a privacy metric
  - An adversary aims to learn which member of an anonymity set (or: group of suspects) performed a certain action (e.g., sent a message)

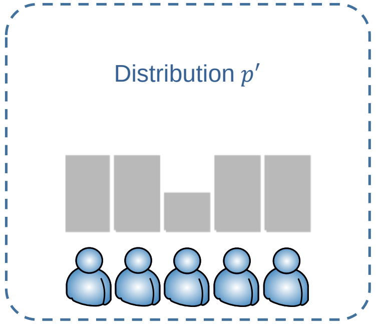
  - Let $\{x_1, x_2, \ldots, x_n\}$ be the anonymity set and $p(x_i)$ the probability estimated by the adversary of x_i being the user who performed such action

  - Attacker's aim: predict the outcome of an r.v. X distributed according to p (identify victim)
  - Defined as $$H(p) = -\sum_i p(x_i) \log p(x_i)$$

System #1



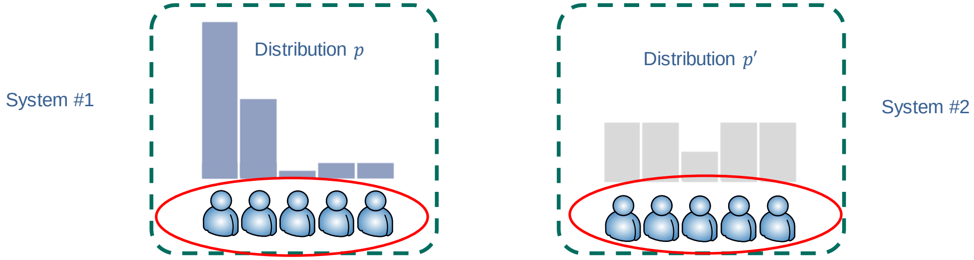
$$H(p) < H(p')$$



System #2

Normalized Shannon's entropy

- SE is useful if the **size** of the anonymity sets of both systems coincide
- Normalized Shannon's entropy allows comparison also otherwise



- What if I tell you that the Shannon's entropy of a system
  - is 4 bits?
  - is 8 bits?

$$H(p) = -\sum_i p(x_i) \log p(x_i) \longrightarrow \frac{H(p)}{\log n}$$

Prove it!

- NSE yields output in (0,1)

Rényi's entropy

- Rényi's entropy is a family of functions widely used in information theory as a measure of uncertainty
- More specifically, Rényi's entropy of order α is defined as

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \sum_{i=1}^{n} p(x_i)^\alpha,$$

support set

$$\begin{cases} H_0(X) = \log \boxed{|\{x \in \mathcal{X} : p(x) > 0\}|} & \text{Hartley} \\ H_1(X) = -\sum_i p(x_i) \log p(x_i) & \text{Shannon} \\ H_\infty(X) = \min_i - \log p(x_i) = -\log \max_i p(x_i) & \text{min-entropy} \end{cases}$$

Interpretation of several entropy measures

- Again, the attacker's aim is to predict the outcome of an r.v. X distributed according to p

$$H_\infty(X) = \min_i -\log p(x_i) = -\log \max_i p(x_i)$$ — worst-case

$$|\wedge$$

$$H_1(X) = -\sum_i p(x_i) \log p(x_i)$$ — average-case

Prove it!

$$|\wedge$$

$$H_0(X) = \log |\{x \in \mathcal{X} : p(x) > 0\}|$$ — best-case

measurement of privacy

Cross-Entropy

- Measurement of the number of bits needed to identify an event *x* drawn from a set *X* if the original data are coded according to the model‘s distribution *P*, not their true distribution *Q*.

$$H(p, q) = -\sum_{x \in \mathcal{X}} p(x)\, \log q(x)$$

- Originated in privacy-preserving ML

2) Information gain/loss-based privacy metrics

- Measure how much information is gained by an adversary after the attack

- Originate from information theory
- Applied to a variety of information, although mostly in anonymous communications and database

- Well-known examples include
  - KL divergence[9]
  - Mutual information[10]
  - Loss of anonymity[11]
  - Information privacy assessment metric (IPAM) [12]

[9] J. Parra-Arnau, D. Rebollo-Monedero, J. Forné, "Measuring the Privacy of User Profiles in Personalized Information Systems", Future Gen. Comput. Syst. (FGCS), vol. 33, pp. 53-63, Apr. 2014.

[10] D. Rebollo-Monedero, J. Forné, J. Domingo-Ferrer, "From t-Closeness-Like Privacy to Postrandomization via Information Theory", IEEE Trans. Knowl., Data Eng., vol. 22, no. 11, pp. 1623-1636, Nov. 2010.

[11] K. Chatzikokolakis, C. Palamidessi, P. Panangaden, "Anonymity protocols as noisy channels", Inf. Comput. 206, 2-4, pp.378-401, Feb. 2008.

[12] S. Oukemeni, H. Rifà-Pous and J. M. Marquès Puig, "IPAM: Information Privacy Assessment Metric in Microblogging Online Social Networks," in IEEE Access, vol. 7, pp. 114817-114836, 2019.

Relative entropy (KL)

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- Given two probability distributions $p(x)$ and $q(x)$ over the same alphabet, the Kullback-Leibler (KL) divergence or relative entropy is defined as

$$D(p \,\|\, q) = E_p \log \frac{p(X)}{q(X)} = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

- Let u denote the uniform distribution on an alphabet of size n. Shannon's entropy is a special case of KL divergence as per

$$D(p \,\|\, u) = \log n - H(p)$$

- Gives a measure of discrepancy between distributions

$$D(p \,\|\, q) \geq 0, \qquad \text{with equality if, and only if, } p = q$$

- Input: prior and posterior distribution of adversary, comp. to true distribution

Interpretation of relative entropy

- We interpret KL divergence as privacy metric in the application of personalized information systems under two different adversary goals
  - Individuation
  - Classification
- Users counter the adversary by distorting their private data

user's actual profile q

user's apparent profile t

Interpretation of relative entropy

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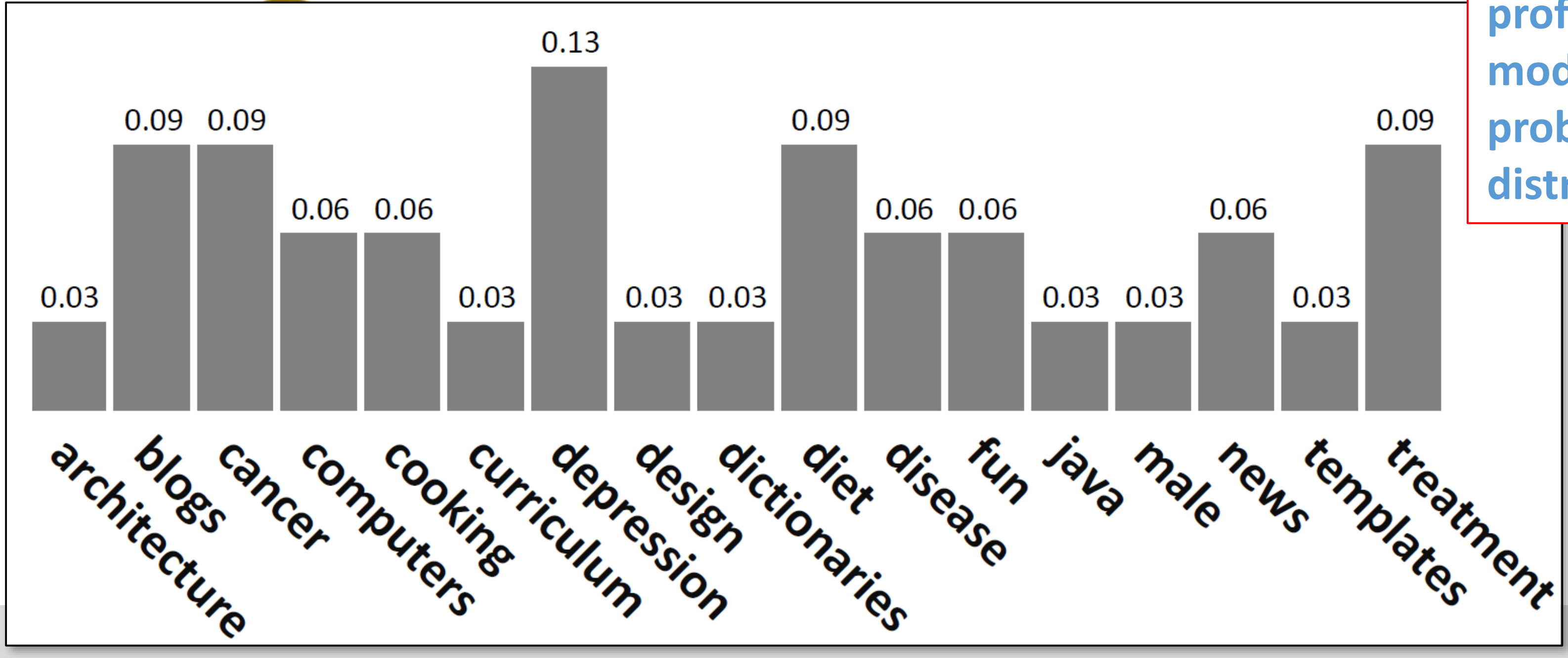
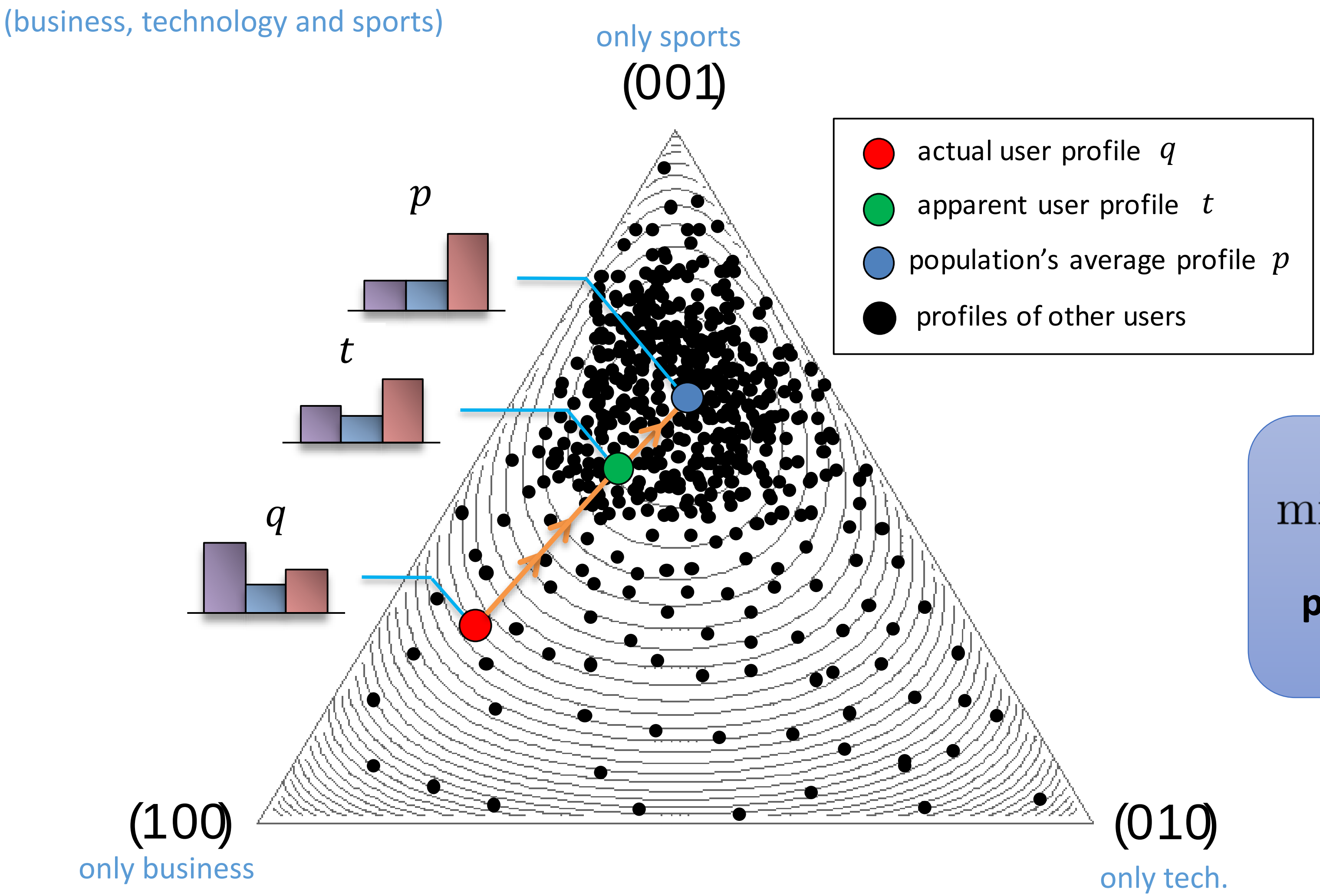


profiles are modeled as probability distributions

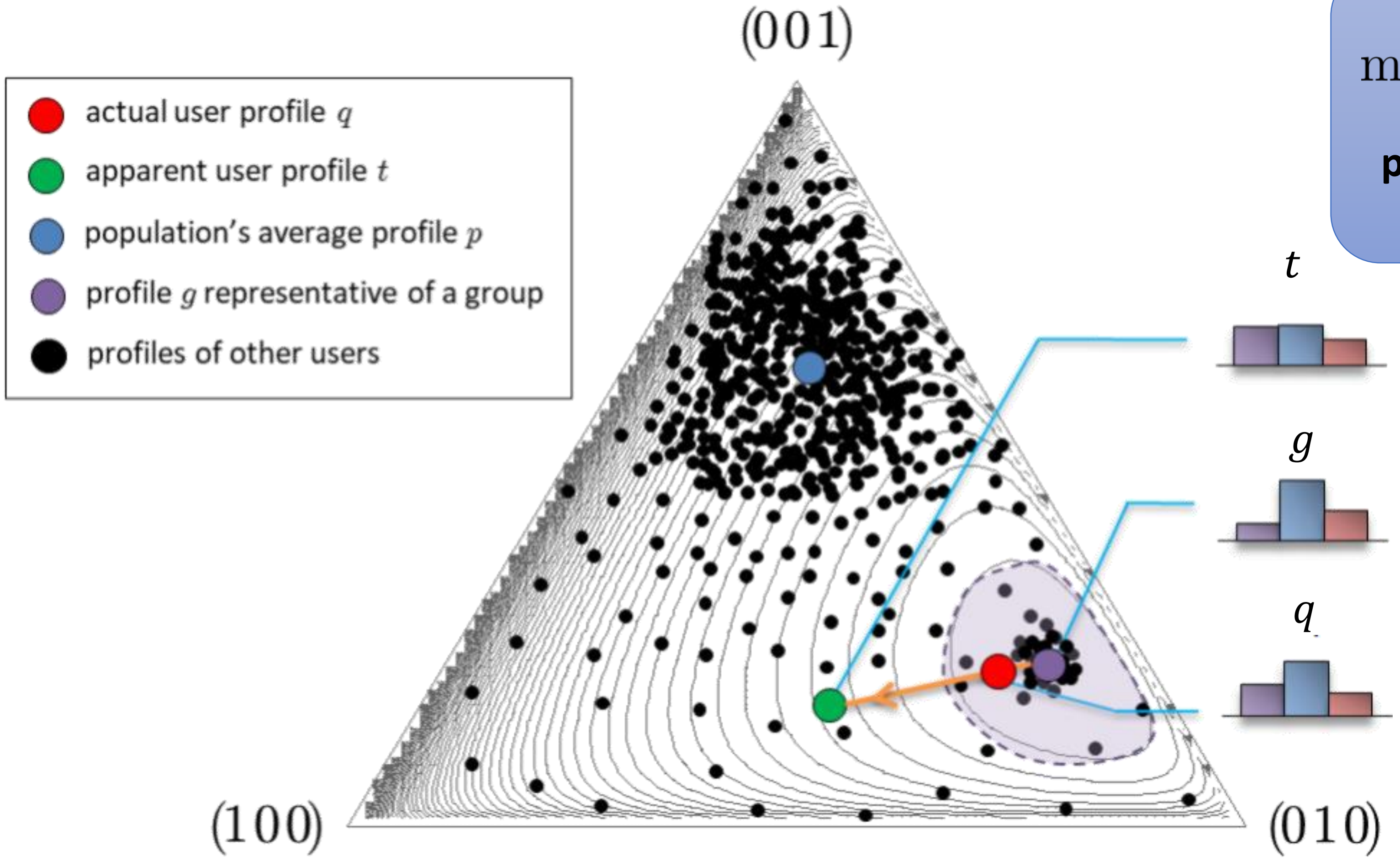
Interpretation of relative entropy



$$\min D(t \,\|\, p)$$

privacy risk

Interpretation of relative entropy



$$\max D(t \,\|\, g)$$

privacy gain

Mutual information

- Consider two random variables X and Y with a joint probability mass function $p(x, y)$ and marginal probability mass functions $p(x)$ and $p(y)$. The mutual information $I(X;\ Y)$ is defined as

$$I(X;Y) = E_{X,Y} \log \frac{p_{X|Y}(X|Y)}{p_X(X)} = D(p_{X,Y} \,\|\, p_X p_Y)$$

- Nonnegativity of mutual information

$$I(X;Y) \geq 0,$$ with equality if and only if X and Y are independent

- Typical use
  - X, sensitive unknown user data
  - Y, data observed by the adversary, accompanied possibly with background-knowledge information; or information disclosed by the user

3) Error-based privacy metrics

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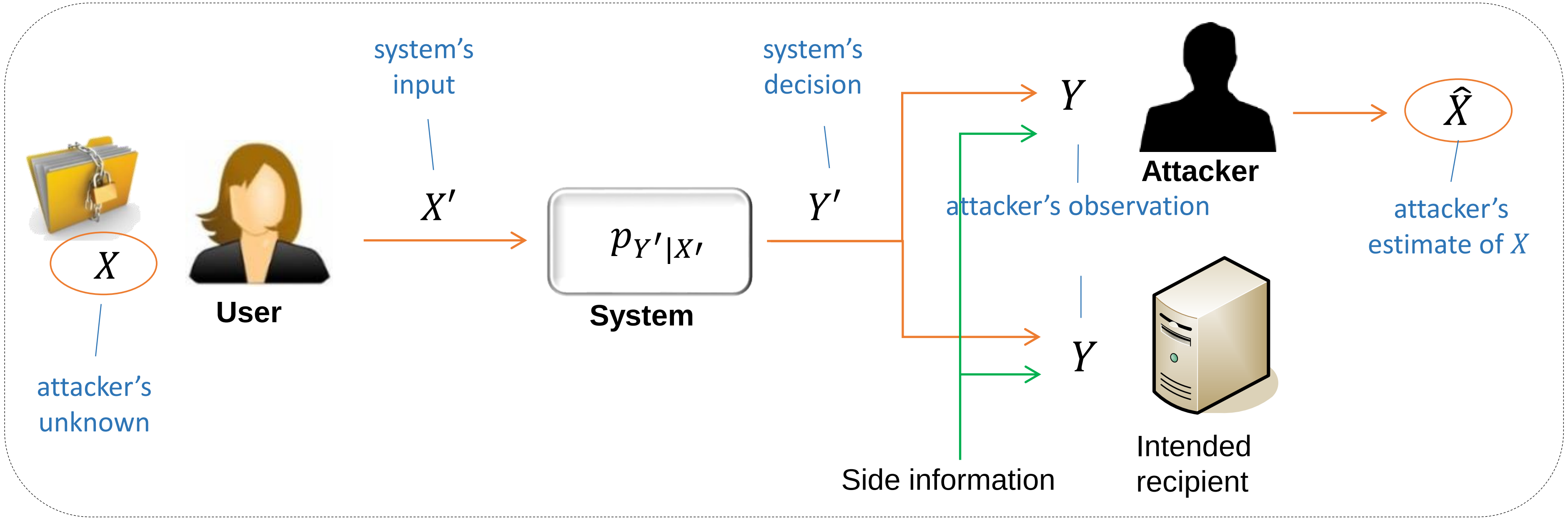
- Measure the error an adversary may make in their attempt to estimate unknown private information
- Examples include
  - Bayes risk – attacker's estimation error by Rebollo et al[13]
  - Correctness, by Shokri et al[14]
  - Mean squared error[15]

[13] D. Rebollo-Monedero, J. Parra-Arnau, C. Diaz, J. Forné, "On the measurement of privacy as an attacker's estimation error", Int. Journal Inform. Secur., vol. 12, no. 2, Apr. 2013, pp. 129-149.

[14] R. Shokri, G. Theodorakopoulos, J.-Y. Le Boudec, J.-P. Hubaux, "Quantifying location privacy", In Proc. IEEE Symp. on Security and Privacy, pp. 247–262, 2011.

[15] S. Oya, C. Troncoso, F. Pérez-González, "Do dummies pay off? Limits of dummy traffic protection in anonymous communications", In Proc. Privacy Enhancing Technologies (PETS), pp. 204–223, 2014.

Attacker's estimation error [13]



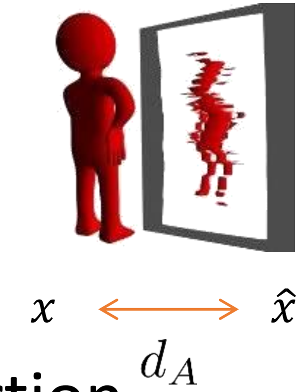
- Probabilistic formulation
  - Confidential information X, unknown to the attacker
  - User's data X' required by the system to make a decision
  - Information disclosed by the system, Y'
  - Information observed by the attacker, Y
  - Attacker's estimate $\hat{X}$ of the confidential information, from observation

Attacker's estimation error[13]

- The attacker's distortion (or error) measure $d_A(x, \hat{x})$ represents the (instantaneous) privacy attained when the unknown confidential information takes on the value $X = x$ but the attacker's estimate is $\hat{X} = \hat{x}$

- We measure privacy as the (expected) privacy attained, also known as Bayes risk,

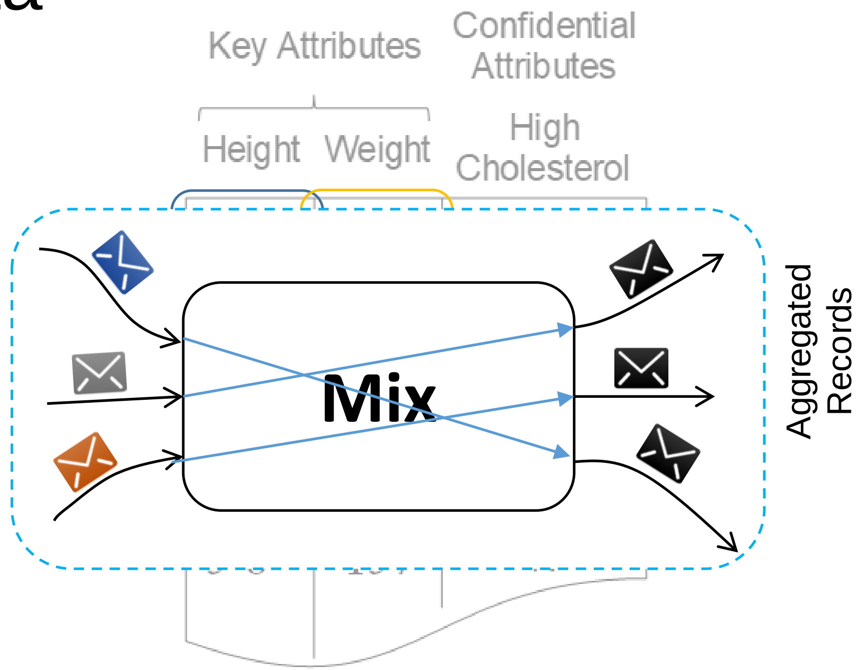
$$P = E\, d_A(X, \hat{X})$$



- Analogously, we measure (expected) utility by using a utility distortion measure $d_S(x', y')$ defined by the system, $D = E\, d_S(X', Y')$

Mean squared error

- What is the most popular measure of utility?
- In microdata



De-identified microdata

Perturbed Key Attributes

Confidential Attributes

Height Weight High Cholesterol

Actual sender	Attacker decision	Squarederror
2	3	1^2
3	1	2^2
1	2	1^2

MSE = (1+4+1)/3 = 2

Protected microdata

[16] S. Oya, C. Troncoso, F. Pérez-González, "Do dummies pay off? Limits of dummy traffic protection in anonymous communications", In Proc. Privacy Enhancing Technologies (PETS), pp. 204-223, 2014.

4) Metrics based on adversary's success probability

- Capture how likely the adversary will be to compromise our privacy in one or several attacks

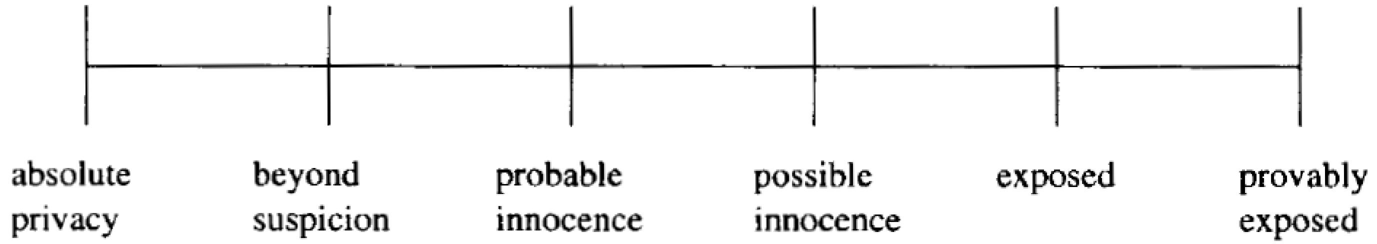
- High privacy correlates with low success probability
- Examples include
  - Degrees of anonymity [17]
  - Sender anonymity [18]

[17] M. K. Reiter and A. D. Rubin, "Crowds: Anonymity for Web transactions", ACM Trans. Inform. Syst. Secur., vol. 1, no. 1, pp. 66-92, 1998.

[18] C. Tripp Barba, L. Urquiza Aguiar, M. Aguilar, J. Parra-Arnau, D. Rebollo-Monedero, J. Forné, E. Pallarès, "A Collaborative Protocol for Anonymous Reporting in Vehicular Ad Hoc Networks", Computer Standards & Interfaces, vol. 36, no. 1, Nov. 2013, pp. 188-197.

Degree of anonymity

- Defined in the context of anonymous communications, with respect to **sender anonymity**



- **Provably exposed**: the attacker can identify (and prove to others) the sender of a message. Formally, $p_1 = 1$.
- **Absolute privacy**: sending a message produces no observable effects on the attacker. Formally, $p_1 = 0$.
- **Beyond suspicion**: the sender appears no more likely to be the originator than others. Formally, $p_1 \leq \mathrm{p}_2, \ldots, \mathrm{p}_\mathrm{n}$
- **Probable innocent**: the sender appears no more likely to be the originator than to not be the originator. Formally, $p_1 \leq 0.5$.
- **Possible innocent**: there is a non-negligible probability that the real sender is someone else. Formally, $p_1 \leq 1 - \delta$, with $\delta \leq 0.5$
- **Exposed:** the adversary's probability is above a threshold τ (e.g., $\tau = 0.9$)

Source: original paper

5) Time-based privacy metrics

- The output is time, an important resource for adversaries to compromise user privacy

- Pessimistically assume the adversary will succeed at some point
  - Time until adversary's success[19]
  - Maximum tracking time[20]

[19] M. Wright, M. Adler, B. N. Levine, C. Shields, "An analysis of the degradation of anonymous protocols", In Proc. Network and Distributed System Security Symp. (NDSS), vol. 2. pp. 39-50, 2002.

[20] K. Sampigethaya, L. Huang, M. Li, R. Poovendran, K. Matsuura, K. Sezaki, "CARAVAN: Providing location privacy for VANET", In Embedded Security in Cars (ESCAR), pp. 29-37, 2005.

Time until adversary's success

- In the context of ACSs
- Measure privacy as the time required for attackers to degrade the anonymity of a particular initiator with high probability
- Define "success"
  - Able to identify n out of N of the target's possible communication peers

Maximum tracking time

- Privacy defined as the cumulative time the attacker tracks a user
- Assumes tracking is carried out only if the size of the anonymity set is 1
- Optimistic or pessimist privacy metric?

6) Data-similarity-based privacy metrics

- Arise in the context of database anonymity

- Measure properties of observable or published data
- Derive the privacy level based on the features of disclosed data

- Well-known examples include
  - k-anonymity[21]
  - p-sensitive k-anonymity[22]
  - l-diversity[23]
  - t-closeness[24]
  - stochastic t-closeness[25]

[21] L. Sweeney, "k-Anonymity: A model for protecting privacy", Int. J. Uncertain., Fuzz., Knowl.-Based Syst., vol. 10, no. 5, pp. 557-570, 2002.
[22] T. M. Truta and B. Vinay, "Privacy protection: p-sensitive k-anonymity property", in Proc. Int. Workshop Priv. Data Manage. (PDM), Atlanta, GA, 2006.
[23] A. Machanavajjhala, J. Gehrke, D. Kiefer, M. Venkitasubramanian, "l-Diversity: Privacy beyond k-anonymity", in Proc. IEEE Int. Conf. Data Eng. (ICDE), Atlanta, GA, Apr. 2006.
[24] N. Li, T. Li, S. Venkatasubramanian, "t-Closeness: Privacy beyond k-anonymity and l-diversity", in Proc. IEEE Int. Conf. Data Eng. (ICDE), Istanbul, Turkey, Apr. 2007, pp. 106-115.
[25] J. Domingo-Ferrer, J. Soria-Comas, "From t-closeness to differential privacy and vice versa in data anonymization", Know.-Based Syst. 74, 1, pp. 151–158, 2015.

k-Anonymity

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Name	DOB	Gender	Zipcode	Disease
Andre	1/21/76	Female	53715	Heart Disease
Beth	4/13/86	Female	53715	Hepatitis
Carol	2/28/76	Male	53703	Brochitis
Dan	1/21/76	Male	53703	Broken Arm
Ellen	4/13/86	Female	53806	Flu
Eric	2/28/76	Female	53806	Hang Nail

Identifying Attribute

Quasi-identifier

Sensitive attribute

a tuple

- The information for each respondent contained in the released data set cannot be distinguished from at least $k - 1$ individuals
- Each tuple of quasi-identifier values in the released table must appear in at least k records

k-Anonymity

date of birth

original table

Name	DOB	Gender	Zipcode	Disease
Andre	1/21/76	Female	53715	Heart Disease
Beth	4/13/86	Female	53715	Hepatitis
Carol	2/28/76	Male	53703	Brochitis
Dan	1/21/76	Male	53703	Broken Arm
Ellen	4/13/86	Female	53806	Flu
Eric	2/28/76	Female	53806	Hang Nail

2-anonymous table

DOB	Gender	Zipcode	Disease
*	Female	5371*	Heart Disease
*	Female	5371*	Hepatitis
*	Male	5370*	Brochitis
*	Male	5370*	Broken Arm
*	Female	538**	Flu
*	Female	538**	Hang Nail

6) Indistinguishability-based privacy metrics

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- Is the adversary able to distinguish between two outcomes of a PET?

- The harder for the adversary to distinguish any pair of outcomes, the higher the privacy provided by the PET
- Typically binary metrics

- Examples include
  - Differential privacy[27]
  - Individual differential privacy[28]

[27] C. Dwork, "Differential privacy," in Proc. Int. Colloq. Automata, Lang., Program. Springer-Verlag, 2006, pp. 1-12.

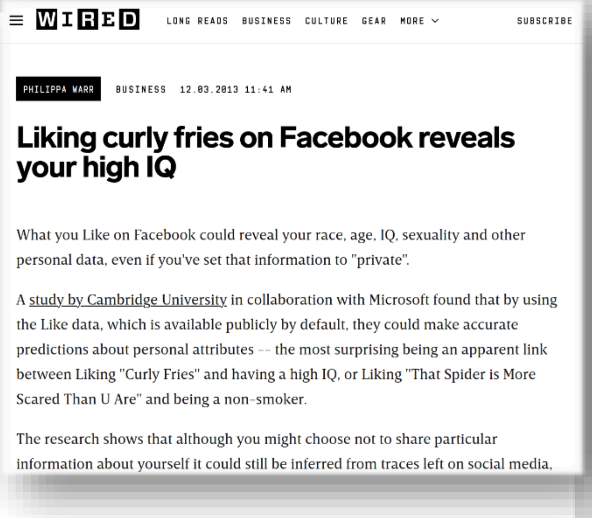
[28] J. Soria-Comas, J. Domingo-Ferrer, D. Snchez, and D. Megas, "Individual differential privacy: a utility-preserving formulation of differential privacy guarantees," IEEE Transactions on Information Forensics and Security, vol. 12, no. 6, pp. 1418–1429, Jun. 2017.

Differential privacy

To be discussed a bit later

- Setting
  - **Database:** composed of individual
  - **Curator:** aimed to protect individua
  - **Analyst or data user:** wishes to perform computations on the database
- A computation protects the privacy of individuals in the data if its output does not reveal any information that is **specific to any individual data subject**
- Differential privacy formalizes this intuition as a mathematical definition

The Inference Privacy Fallacy


≡ WIRED LONG READS BUSINESS CULTURE GEAR MORE SUBSCRIBE

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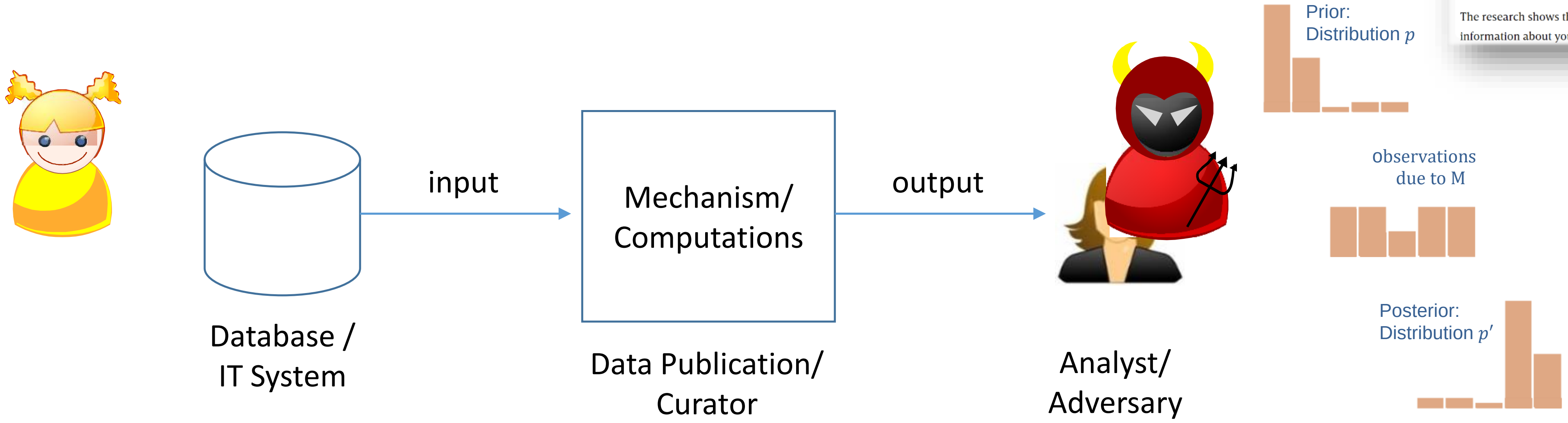
Liking curly fries on Facebook reveals your high IQ

What you Like on Facebook could reveal your race, age, IQ, sexuality and other personal data, even if you've set that information to "private".

A study by Cambridge University in collaboration with Microsoft found that by using the Like data, which is available publicly by default, they could make accurate predictions about personal attributes -- the most surprising being an apparent link between Liking "Curly Fries" and having a high IQ, or Liking "That Spider is More Scared Than U Are" and being a non-smoker.

The research shows that although you might choose not to share particular information about yourself it could still be inferred from traces left on social media.

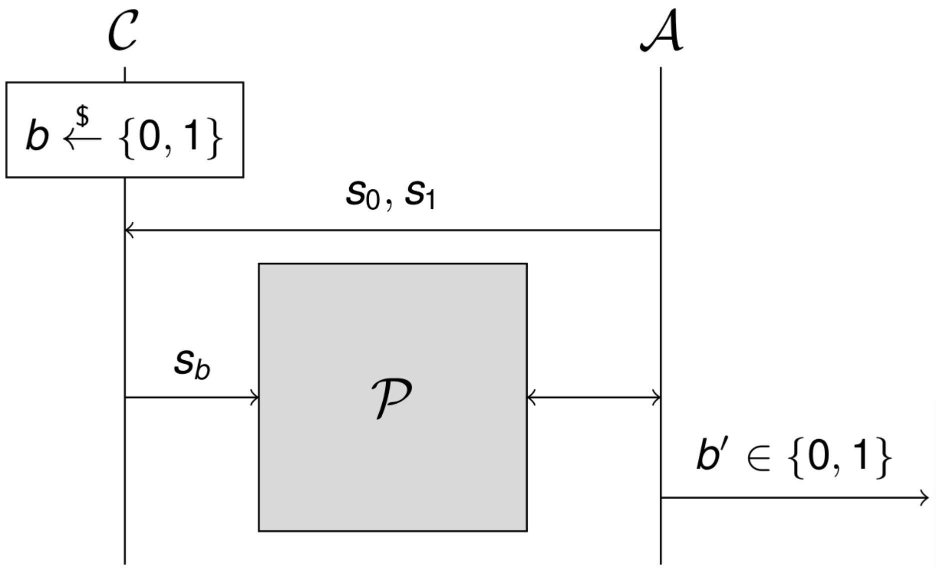
- We measure the privacy of the data release mechanism



- We cannot protect adaptation of the prior (and corresponding inference)
- General: If statistics are revealed, they are useless or help improve the prior

Back to ACS: Indistinguishability Games

- Recall IND-CPA game from crypto…



[29] Kuhn et al., "On Privacy Notions in Anonymous Communication", PoPETS (2) 2019: 105-125

- Communication properties
  - U and U' – Which senders/receivers are active?
  - |U| and |U'| – How many senders/receivers are active?
  - Q and Q' – Which user sends/receives how many messages?
  - H and H' – How many users sends/receives how many messages?
  - P and P' – Which messages are send/received by the

Example: Sender Notions

- All disclose receiver-message relation, but hide who sends which message
- **Sender Unobservability (S$\overline{O}$)** additionally discloses number of
- communications
- **Sender-Frequency Unlinkability** (SF$\overline{L}$) additionally discloses number of communications and set of active users
- **Sender-Messages Unlinkability** (SM$\overline{L}$) additionally discloses number of communications, set of active users, and number of messages per sender

Summary

- Selection of over 25 privacy metrics across four privacy domains
- Followed the structure proposed by[1] based on metrics' outputs
  - Uncertainty
  - Information gain/loss
  - Error-based metrics
  - Success-estimate metrics
  - Time-based metrics
  - Data similarity/dissimilarity
  - Indistinguishable
- Best-case (optimisic), average-case, and worst-case (pessimistic)
- Connections among them

[1] Isabel Wagner and David Eckhoff, "Technical Privacy Metrics: A Systematic Survey", ACM Comput. Surv. 51, 3, Article 57, June 2018.